LMM for a Nonlinear Model of Convective Straight Fins with Variable Thermal Conductivity and Heat Transfer Coefficient

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ABSTRACT: The nonlinear problem of convective straight fins with temperature-dependent thermal conductivity and heat transfer coefficient has been tackled by the use of Leibnitz-Maclaurin Method (LMM) via Successive Differential Coefficients (SDC), which is a power series solution technique. Also, an exact implicit integral solution was constructed for the problem. The obtained solutions satisfy the physical boundary conditions. Benchmarks or validation tests of the LMM and the exact implicit integral solution together with numerical experiments demonstrated excellent agreements. Parametric analyses indicated that the thermal conductivity parameter β, thermo-geometrical property ψ, and the heat transfer mode m of fins characterize the key elements for the numerical computations and descriptions of the fin tip temperatures θ_w, fin base gradient temperature $\theta'(1)$, and fin efficiency η. The variability of the thermal conductivity and heat transfer coefficient demonstrated the physical usefulness and serves as a practical departure from the previously investigated constant scenarios of thermal conductivity and heat transfer coefficient.

Keywords: Exact Implicit Integral Solution, Leibnitz-Maclaurin Method (LMM), Nonlinear Problem, Straight Fins, Success Differential Coefficients (SDC).

1. INTRODUCTION

Fins are highly conductive metallic surfaces that are used to increase the heat transfer of heating systems such as car radiators and heating units, heat exchangers, air-cooled engines, electrical transformers, motors, electronic transistors, refrigeration, cooling of oil carrying pipe, cooling electric transformers, cooling of computer systems and air conditioning. Other applications are petrochemical plants, gas treatment plants, natural gas liquefaction plants, air separationplants, helium liquefaction plants, etc. In nature, the ears of rabbit act as fins to release heat from the blood. Due to the vast applications of fins, experimental and theoretical studies of fins have dated back several years. Among others, many researchers had devoted efforts to fin design optimization and cost effectiveness for the end need to increase or enhance the rate of heat transfer. Kraus et al. [1] had addressed important design guidelines for cooling electronic devices, and there are documentations of several fin surfaces and their characteristics [2, 3]. Thereare lots of literature in fins with novel contributions and various solution methods to the governing mathematical models.

The mathematical models resulting from the energy balance of finned surfaces are either partial or ordinary differential equations for the temperature distribution; depending also on whether the heat transfer scenarios are steady or unsteady states. It is important to state that most of the mathematical problems resulting from natural and scientific phenomena are non-linear. It is also known that quite a good number of the applications of fins use variable or temperature-dependent thermal conductivity and heat transfer coefficient. This is because fins with variable thermal conductivity and heat transfer coefficient are more physically realistic, which are as a result of large temperature difference that exists within the fins. The resulting non-linear problem gives a unified model equation for the heat transfer mechanisms of conduction, convection, thermal radiation, nucleate boiling and many more other heat transfer modes, which are discussed in Holman [4], Moitsheki et al. [5], Ganji and Dogonchi [6]. It must be said here that finding exact analytical results for non-linear equations are quite elusive, if not impossible. Therefore, many thermal engineers and researchers have resorted to using approximate or semi-analytical solutions and numerical experiments in order to gain insights. The semi-analytical methods such as the perturbation method (PM), homotopy perturbation method (HPM), variational iteration method (VIM), homotopy analysis method (HAM), decomposition method (DM) and differential transform method (DTM) are powerful mathematical techniques that proffer solutions for both linear and non-linear problems. The literature is rife in the use of these and other techniques in the solutions and analyses of fin problems. Recently, Mebine and Olali [7] used Leibnitz-Maclaurin Method (LMM) via Successive Differential Coefficients (SDC)to solve the problem of convective straight fins with temperature-dependent thermal conductivity. It was observed that there was an excellent performance of the LMM when compared to the results of DTM, HPM and HAM.

The major concern of this work is of two-fold. Firstly, it is complementary to the work of Mebine and Olali [7]. Secondly, the work is aimed to further validate the LMM by consideringboth temperature-dependent thermal conductivity and heat transfer coefficient to enhance the investigation of other heat transfer mechanisms such as laminar film boiling or condensation, laminar natural convection, turbulent natural convection, nucleate boiling and radiation in a unified temperature distribution model in straight fins. The variability of thermal conductivity and heat transfer coefficient is physically meaningful and is intendedto serve as a practical departure from the previously investigated temperature-dependent thermal conductivity and constant heat transfer coefficient.The sections that follow hereafter are the Mathematical Formulations, Integral and some Exact Results, Leibnitz-Maclaurin Method of Solution, Analyses of Results, and Concluding Remarks.

2. MATHEMATICAL FORMULATIONS

Consider a straight fin with a temperature-dependent thermal conductivity and heat transfer coefficient, with an arbitrary constant cross sectional area A_C; perimeter P and length b. The fin is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a, and its tip is insulated. The one-dimensional energy balance equation is given as follows:

$$A_C \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - Ph(T)\,(T_b - T_a) = 0\,, \qquad (1)$$

where T is temperature, $k(T)$ is the temperature-dependent thermal conductivity of the fin material, P is the fin perimeter, and $h(T)$ is the temperature-dependent heat transfer coefficient. The temperature-dependent thermal conductivity and the heat transfer coefficient of the material are respectively assumed as follows:

$$k(T) = k_a[1 + \lambda(T - T_a)], \qquad (2)$$

$$h(T) = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^m, \quad (3)$$

where k_a is the thermal conductivity at the ambient fluid of the fin temperature, while h_b is the heat transfer coefficient at the fin base, λ is the parameter describing the variation of the thermal conductivity and m is a constant that expresses the nonlinearity of the variability of the heat transfer coefficient, and some typical values of it physically signifyvarious heat transfer modesor mechanisms and may vary from -6.6 to 5. However, in most practical applications the exponent m lies between -3 and 3. Some typical values of the exponent m are $-\frac{1}{4}$ represents laminar film boiling or condensation, $\frac{1}{4}$ for laminar natural convection, $\frac{1}{3}$ for natural turbulent convection, 2 for nucleate boiling, 3 for radiation and 0 implies constant heat transfer coefficient. Unal [8] stated that exact solutions may be constructed for the steady-state one-dimensionaldifferential equation describing temperature distribution in a straight fin when the thermal conductivity is a constant and $m = -1, 0, 1$ and $m = 2$.

The temperature-dependent thermal conductivity and heat transfer coefficient equations (2) and (3) are deemed fit for many industrial or engineering applications. The appropriate dimensionless parameters are

$$\theta = \frac{T - T_a}{T_b - T_a}, \xi = \frac{x}{b}, \beta = \lambda(T_b - T_a), \psi = \left(\frac{h_b P b^2}{k_a A_C} \right)^{1/2}. (4)$$

Equation (1) is now rendered dimensionless as

$$\frac{d^2\theta}{d\xi^2} + \beta\theta \frac{d^2\theta}{d\xi^2} + \beta \left(\frac{d\theta}{d\xi} \right)^2 - \psi^2 \theta^{m+1} = 0. (5)$$

The associated dimensionless boundary conditions are now written as

$$\frac{d\theta}{d\xi}=0 \quad \text{when} \quad \xi=0$$
$$\theta=1 \quad \text{when} \quad \xi=1, \qquad (6a,b)$$

where θ is the dimensionless temperature, ξ is the non-dimensional coordinate, β is the non-dimensional parameter describing thermal conductivity, and ψ is the thermo-geometric fin parameter. The set of equations (5, 6) are investigated by various researchers such as Khani et al.[9]applied HAM and Kim et al. [10] utilized ADM and Taylor transformation. It was observed in Khani et al. [9] that ADM and HPM solutions fail when ψ increases to a large number, but HAM solutionremained accurate.

On the other hand, when only the thermal conductivity varies with temperature and the heat transfer coefficient becomes a constant (i.e. $m=0$), this situation has been solved with some semi-analytical methods, such as LMM [7], PM [11], HPM [11, 12], VIM [11,13], HAM [12,14], DTM [15] and ADM [16].

The heat transfer rate from the fin is found by using Newton's law of cooling which states that "for a body cooling in a draft, that is, forced convection, the rate of heat loss is proportional to the difference in temperature between the body and the surrounding."

Consider

$$Q=\int_0^b P(T-T_a)dx. \qquad (7)$$

The ratio of the fin heat transfer rate to the heat transfer rate of the fin if the entire fin was at the base temperature is commonly known as the fin efficiency:

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T-T_a)dx}{Pb(T_b-T_a)} = \int_{\xi=0}^{1} \theta^{m+1} d\xi . \quad (8)$$

In other words, the fin efficiency is simply the parameter that indicates the effectiveness of a fin in transferring a given quantity of heat.

The equations (5) together with the equations (6) are solved with the LMM via the SDC for the analyses of the problem of fin efficiency (8) of convective straight fins with temperature-dependent thermal conductivity and heat transfer coefficient.

3. INTEGRAL AND SOME EXACT SOLUTIONS

As a springboard to constructing an integral solution, the first derivative appearing in the governing equation (5) is eliminated, which transforms slightly the equation to

$$\frac{d^2\theta}{d\xi^2} + \frac{1}{2}\beta\frac{d^2\theta^2}{d\xi^2} - \psi^2\theta^{m+1} = 0 . \qquad (9)$$

Now multiplying the equation (9) by $\frac{d\theta}{d\xi}$ and integrating gives the general integral solution

$$\xi = \int_{\theta_w}^{\theta} \frac{\sqrt{(m+2)(m+3)}\,(1+\beta\zeta)}{\sqrt{2\psi^2\left[\beta(m+2)\,\zeta^{m+3} + (m+3)\,\zeta^{m+2} - \beta(m+2)\theta_w^{m+3} - (m+3)\theta_w^{m+2}\right]}}\, d\zeta \;,(10)$$

where θ_w defines the temperature at $\xi = 0$, which could be computed with the help of the condition $\theta(1) = 1$ by the equation

$$1 = \int_{\theta_w}^{1} \frac{\sqrt{(m+2)(m+3)}\,(1+\beta\zeta)}{\sqrt{2\psi^2\left[\beta(m+2)\,\zeta^{m+3} + (m+3)\,\zeta^{m+2} - \beta(m+2)\theta_w^{m+3} - (m+3)\theta_w^{m+2}\right]}}\, d\zeta \;. \qquad (11)$$

Equation (11) shows the connection between the temperature θ_w at the fin tip and the thermo-geometric parameter ψ. From the physical point of view, $\theta(0)=\theta_w$ is the quantity of interest, and it is the rate of heat at the tip of the fin.Equation (10) is an exact implicit solution andin general, physically significant for numerical integration, except for $m \neq -2$ and $m \neq -3$.Without loss of generality, by reason of the condition $\theta(1)=1$, it may be inferred from equation (10) that

$$\theta'(1)=\frac{\sqrt{2\psi^2\left[\beta(m+2)+(m+3)-\beta(m+2)\theta_w^{m+3}-(m+3)\theta_w^{m+2}\right]}}{\sqrt{(m+2)(m+3)}\,(1+\beta)}. \quad (12)$$

Equation (12) is a general relationship that connects the temperature gradient at the base of the fin $\theta'(1)$ to the temperature θ_w at the tip of the fin, the variable thermal conductivity parameter β, the thermo-geometric parameter ψ, and the heat transfer mode m.Equation (12) is acceptableexcept for $m \neq -2, -3$ and $\beta \neq -1$. Similar derivations to equations (10), (11) and (12) are equally deduced by Latiff et al. [17] using symmetry reduction methods.

Other exact results for constant thermal conductivity $\beta = 0$ and $m \neq -2$ derived by Min-Hsing [18], Abbasbandy and Shivanian [19], and Haq and Ishaq [20] are

$$\xi=-\frac{\sqrt{2m+4}}{m\psi}\theta^{-\frac{m}{2}}{}_2F_1\left(\left[\frac{1}{2},\frac{m}{2m+4}\right],\left[\frac{3}{2}-\frac{1}{m+2}\right],\left(\frac{\theta}{\theta_w}\right)^{-m-2}\right)-\frac{1}{\psi}\sqrt{\pi}\sqrt{\frac{m}{2}+1}\,\theta_w^{-\frac{m}{2}}\frac{\Gamma\left(\frac{m}{2m+4}\right)}{\Gamma\left(-\frac{1}{m+2}\right)}, \quad (13)$$

$$\psi=-\frac{\sqrt{2m+4}}{m}{}_2F_1\left(\left[\frac{1}{2},\frac{m}{2m+4}\right],\left[\frac{3}{2}-\frac{1}{m+2}\right],\theta_w^{m+2}\right)-\frac{1}{\psi}\sqrt{\pi}\sqrt{\frac{m}{2}+1}\,\theta_w^{-\frac{m}{2}}\frac{\Gamma\left(\frac{m}{2m+4}\right)}{\Gamma\left(-\frac{1}{m+2}\right)}, \quad (14)$$

where ${}_2F_1$ is the hypergeometric function as represented in Abramowitz and Stegun [21]:

$$ {}_2F_1([a,b],[c],x) = \frac{\Gamma(c)}{\Gamma(b)\,\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-t\ x)^{-a}\, dt\,. $$

Specifically, for constant thermal conductivity $\beta = 0$, the following exact results also hold:

$$ \xi = \frac{1}{\psi}\sqrt{\frac{\pi}{2}}\ \mathrm{erfi}\left(\sqrt{\mathrm{Log}\left(\frac{\theta}{\theta_w}\right)}\right)\theta_w \quad \text{for} \quad m = -2\,, \qquad (15) $$

$$ \psi = \sqrt{\frac{\pi}{2}}\ \mathrm{erfi}\left(\sqrt{\mathrm{Log}\left(\frac{1}{\theta_w}\right)}\right)\theta_w \quad \text{for} \quad m = -2\,, \qquad (16) $$

$$ \theta = \frac{1}{2}\psi^2\xi^2 - \frac{1}{2}\psi^2 + 1 \quad \text{for} \quad m = -1, (17) $$

$$ \theta = \frac{\cosh(\psi\xi)}{\cosh(\psi)} \quad \text{for} \quad m = 0, (18) $$

where $\mathrm{erfi}(z) = \int_0^z \exp(-t^2)dt$. Equations (15, 16) are also reported in Abbasbandy and Shivanian [19]. It is worth stating here that the exact results are vital for the purpose of comparisons with the LMM and numerical computations.

4. LEIBNITZ-MACLAURIN METHOD OF SOLUTION

For two differentiable and continuous functions, say u and v, which are functions of say x, Leibnitz concise formula for the nth differential coefficient of their product is:

$$ D^n(uv) = \sum_{r=0}^{n} {}^nC_r D^{n-r}v \cdot D^r u\,, \qquad (19) $$

where ${}^nC_r = \frac{n!}{(n-r)!r!}, D = \frac{d}{dx}, D^2 = \frac{d^2}{dx^2}, D^3 = \frac{d^3}{dx^3}, \ldots, D^n = \frac{d^n}{dx^n}$.

The Leibnitz's formula is applied to the differential equations (5, 6) in obtaining a recurrence relation between successive differential coefficients. This forms a step towards finding a power series solution of the problem at hand.

In the application of Leibnitz's formula, the solution of the problem is written in terms of Maclaurin series which is a special case of Taylor series, such that

$$\theta(\xi) = \theta(0) + \frac{\xi}{1!}\theta'(0) + \frac{\xi^2}{2!}\theta''(0) + \ldots = \sum_{r=0}^{n} \frac{\xi^r}{r!}\theta^{(r)}(0), \qquad (20)$$

where $\theta(0) = \theta(\xi)\big|_{\xi=0}, \theta'(0) = \frac{d\theta}{d\xi}\big|_{\xi=0}, \theta''(0) = \frac{d^2\theta}{d\xi^2}\Big|_{\xi=0}, \cdots, f^{(r)}(0) = \frac{d^r\theta}{d\xi^r}\big|_{\xi=0}$.

From the boundary conditions, it is observed that the value of $\theta(0) = \theta_w$ is unknown as it is in the integral solution (10), which would be computed with the help of condition $\theta(1) = 1$. The recurrence relation for the SDC by the application of equation (19) to the equation (5) is now written as

$$\theta^{n+2}(0) = \psi^2 \sum_{r=0}^{n} {}^nC_r \; D^{n-r}(1+\beta\,\theta(0))^{-1} \cdot D^r(\theta(0))^{m+1} - \beta \sum_{r=0}^{n} D^{n-r}(1+\beta\,\theta(0))^{-1} \cdot D^r\left(\frac{d\theta(0)}{d\xi}\right)^2 \text{ for } n \geq 0. \quad (21)$$

The associated initial conditions are

$$\theta(0) = \theta_w, \; \theta'(0) = 0. \qquad (22)$$

Equation (21) constitutes a system of equations for the coefficients $\theta''(0), \theta'''(0), \theta^{(iv)}(0), \cdots, \theta^{(r+1)}(0)$, which could be solved for any particular heat transfer mode m. With the equation (21) subject to the initial conditions (22), one can readily and easily compute recursively the first few SDC with pen on paper! One advantage of the SDC is that in some physical problems only few terms may be computed and it converges to the required result. Of course, with the aid of Symbolic Computation Software such as Maple, Mathematica and Matlab, as many terms as possible and as desired could be computed! It is important to note that the Taylor series and the Maclaurin series only represent the function $\theta(\xi)$ in their intervals of convergence.

Apart from using the exact implicit integral solution (10) and the other exact solutions (13 - 18), all herein referred to asEXACT aimed at the validation of the LMM solutions (20), a Fifth-order *Runge-Kutta-Fehlberg* numerical solution method implemented in MAPLE, herein referred to as NUM, is also utilized in this work.

5. ANALYSES OF RESULTS

The basic physically important parameters entering the problem arethe temperature gradient at the base of the fin $\theta'(1)$, the temperature θ_w at the tip of the fin, the fin efficiency η, the variable thermal conductivity parameter β, thermo-geometric parameter ψ, and the heat transfer mode m. To discuss the effects of these parameters, tabular and graphical representations are made using respective values of β, ψ, and m for the computations of θ_w, $\theta'(1)$, and η.

Table 1 displays the temperature θ_w at the tip of the finfor various heat transfer modes m with constant thermal conductivity. The LMM demonstrated excellent agreement with the NUM and EXACTresults. It is important to note here that the LMM, NUM and EXACT results have been kept at six-decimal places without rounding-up, whereas the DTM [22] results have been rounded-up. This accuracy gives high confidence in the validity of the LMM, and reveals its power of accountable judgement to engineering problems within its radius of convergence. It is observed from the table that the values of the temperature at the tip of the fin increases with increase in the heat transfer mode for $\beta = 0, \psi = 0.5$, while the temperature at the tip of the fin due to $\beta = 0, \psi = 1.0$

decreases as m increasesfrom -3 to -1, and increases from -1 to 3. Physically, the thermo-geometric parameter ψ actually accounts for convective-conductive effects [22] in the fin system.

Table 2 accounts for the effect of variable thermal conductivity that it increases the wall temperature. The other observations made in Table 1 are equally seen manifesting in Table 2.

	(a) $\beta=0, \psi=0.5$				(b) $\beta=0, \psi=1.0$			
m	**LMM**	**NUM**	**EXACT**	**DTM [22]**	**LMM**	**NUM**	**EXACT**	**DTM [22]**
-3	0.830016	0.830016	0.830016	-	0.885959	-	-	-
-2	0.858166	0.858166	0.858166	0.858211	0.727155	-	-	-
-1	0.875000	0.875000	0.875000	0.875000	0.500000	0.500000	0.500000	0.500000
0	0.886818	0.886818	0.886818	0.886819	0.648054	0.648054	0.648054	0.648054
1	0.895803	0.895803	0.895803	0.895804	0.712256	0.712256	0.712256	0.712258
2	0.902973	0.902973	0.902973	0.902974	0.751622	0.751622	0.751622	0.751635
3	0.908888	0.908888	0.908888	0.908889	0.779145	0.779145	0.779145	0.779177

Table 1: Heat transfer at the tip of the fin $\theta(0)$ with various heat transfer modes m for constant thermal conductivity with thermo-geometric fin parameter (a) $\beta=0$, $\psi=0.5$; (b) $\beta=0$, $\psi=1.0$

	(a) $\beta = 0.1, \psi = 0.5$			(b) $\beta = 0.5, \psi = 1.0$		
m	**LMM**	**NUM**	**EXACT**	**LMM**	**NUM**	**EXACT**
-3	0.850537	0.850537	-	0.821447	-	-
-2	0.872005	0.872005	-	0.671272	-	-
-1	0.885770	0.885770	0.885770	0.645751	0.645751	0.645751
0	0.895757	0.895757	0.895757	0.729675	0.729675	0.729675
1	0.903502	0.903502	0.903502	0.772484	0.772484	0.772484
2	0.909768	0.909768	0.909768	0.800260	0.800260	0.800260
3	0.914988	0.914988	0.914988	0.820310	0.820310	0.820310

Table 2: Heat transfer at the tip of the fin $\theta(0)$ with various heat transfer modes m for variable thermal conductivity with thermo-geometric fin parameter (a) $\beta = 0.1$, $\psi = 0.5$; (b) $\beta = 0.5$, $\psi = 1.0$

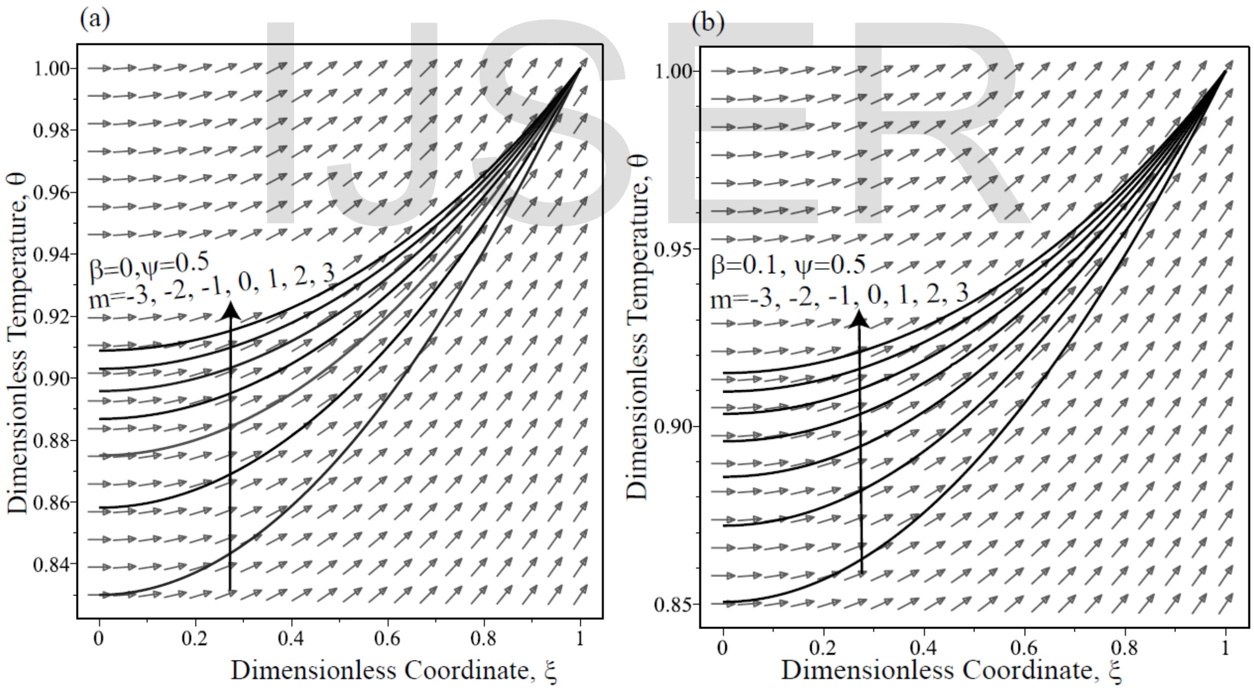


Figure 1: Dimensionless temperature θ versus dimensionless coordinate ξ for various heat transfer modes m for (a) $\beta = 0, \psi = 0.5$; (b) $\beta = 0.1, \psi = 0.5$

Figure 1 depicts two scenarios-solid lines and field plots (arrows). The solid lines are plots of the non-dimensional temperature θ versus the non-dimensional coordinate ξ for various heat transfer modes m for (a) $\beta = 0, \psi = 0.5$; (b) $\beta = 0.1, \psi = 0.5$, while the field plots or direction fields (the arrows) are for the particular cases of $\psi = 0 \cdot 50$ when $\beta = 0$ and $\psi = 0 \cdot 50$ when $\beta = 0.1$ with $m = 0$, which are visualizations of $\theta(0) = 0.886818$ and $\theta(0) = 0.895757$, respectively. These typify respectively the rate of heat at the wall in the absence of thermal conductivity parameter $\beta = 0$ and the presence of thermal conductivity parameter $\beta = 0.1$. No doubt, it is observed that a little increase in the thermal conductivity parameter β increases the wall temperature. Physically, thermal conductivity enhances the wall temperatures. From the Figure 1, it is equally seen that increase in the wall temperature $\theta(0)$ depends on the heat transfer mode value m. Physically, it could be inferred that the need for the use of any particular heat transfer mode depends on the user in relation to the thermo-geometric ψ and thermal conductivity β parameters.

The equation (12), which gives a general relationship among the fin tip temperature θ_w, the fin base gradient temperature $\theta'(1)$, the mode of heat transfer m, the thermo-geometric ψ and thermal conductivity β parameters are shown in Figure 2. The dashed lines are due to equation (12), while the points are the LMM solution. It is seen that the LMM result fits well the implicit exact solution (12). The Figure 2 shows that the fin tip temperature decreases to zero when the magnitude of the fin base gradient temperature increases, and the rate of decay to zero decreases with increasing $\theta'(1)$ when the value of m decreases.It can also readily be observed from Figure 2 that the thermo-geometric ψ enhances the fin base gradient temperature $\theta'(1)$ at the absence of the thermal conductivity β parameter, while the presence of the thermal conductivity β parameter decreases the fin base gradient temperature $\theta'(1)$ as the heat transfer mode m increases. Of the course, the maximum value of the fin tip temperature θ_w is 1.

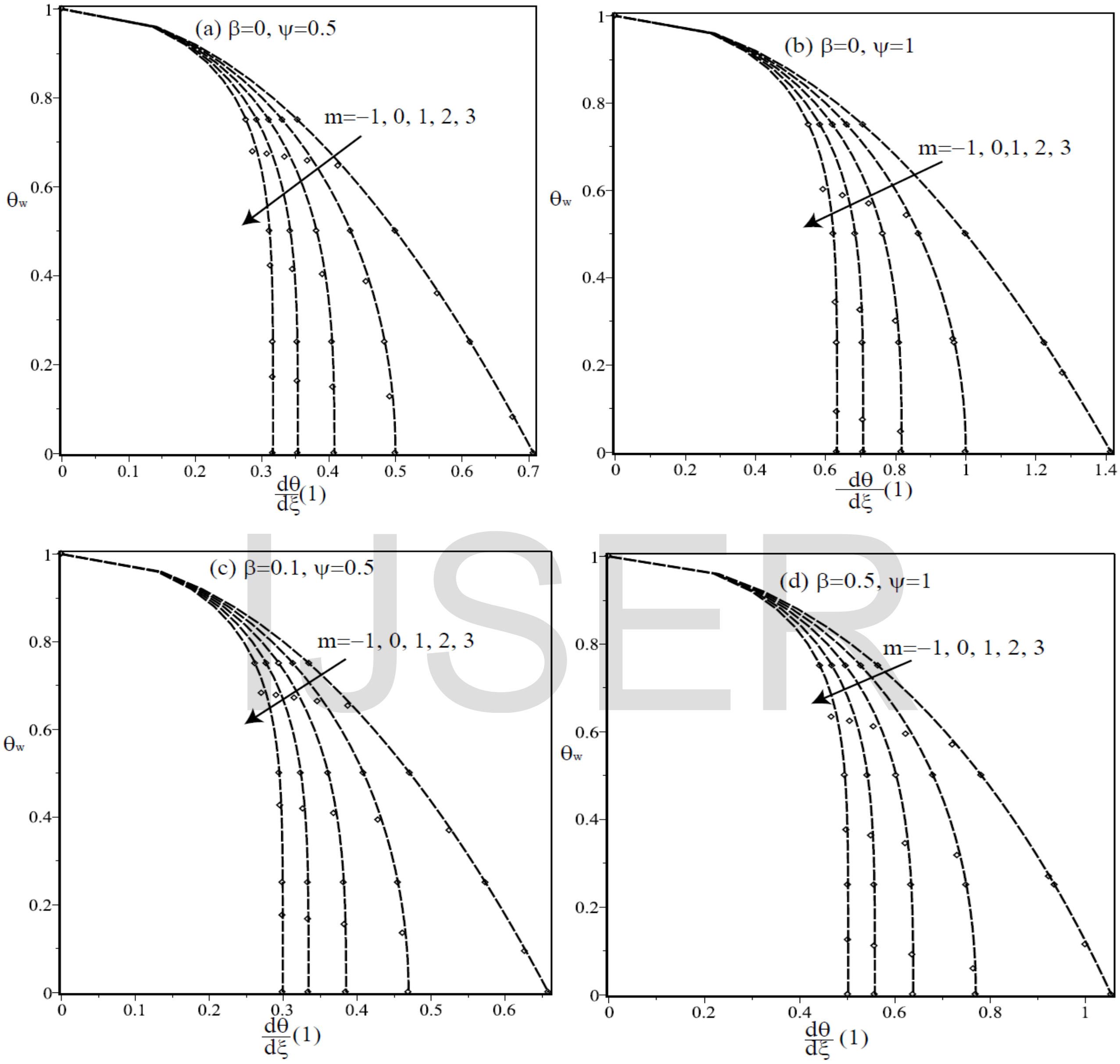
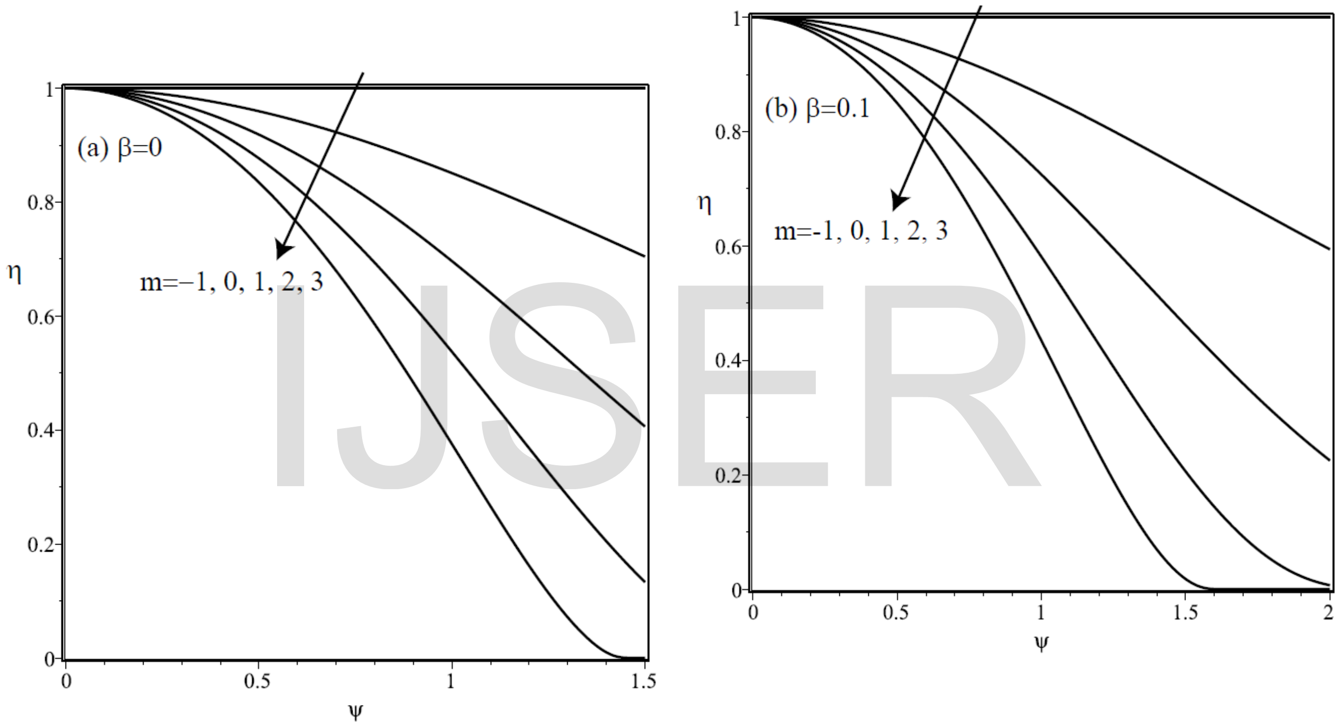


Figure 2: Fin tip temperature θ_w versus fin base temperature gradient $\theta'(1)$ for various heat transfer modes m for (a) $\beta = 0, \psi = 0.5$; (b) $\beta = 0, \psi = 1$; (c) $\beta = 0.1, \psi = 0.5$; (d) $\beta = 0.5, \psi = 1$

One of the most important characteristics in the study of fins in engineering applications is fin efficiency. The fin efficiency η for several heat transfer modes m with the thermo-geometric parameter ψ are displayed in Figure 3 for various values of the thermal conductivity parameter β.

The results show that $m = -1$, which indicates a uniform local heat flux over the whole fin surface gives the result of $\eta = 1$ as clearly indicated for all values of β in the Figure 3. It should be emphasized here that no physical system in real world gives 100 percent efficiency throughout! The LMM resultsfor η are, only valid for $m \geq -1$ as displayed in Figure 3. It is observed that for $m > -1$, the fin efficiency η decreases with increasing ψ or m, and while the value of β increases the boundary layer of ψ. Obviously, ψ and β play important roles in describing the efficiency of heat loss of fins in relation to the particular mode of heat transfer m.



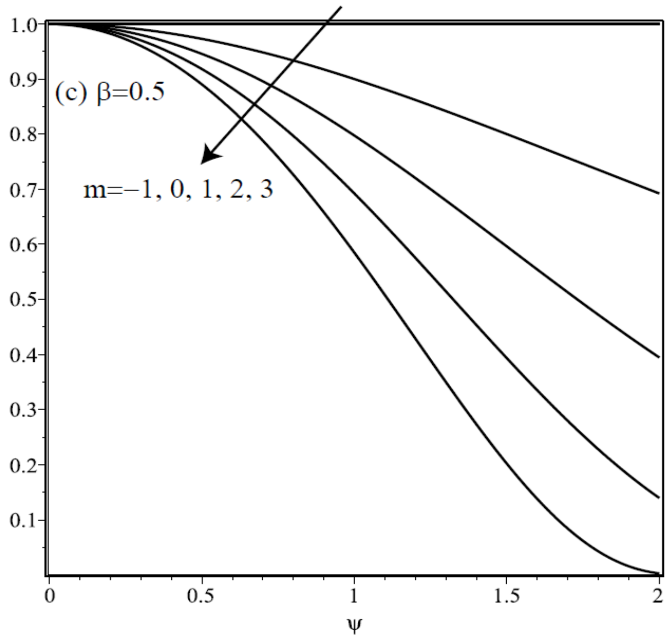


Figure 3: Fin efficiency versus ψ for various heat transfer modes m for (a) $\beta = 0$; (b) $\beta = 0.1$; (c) $\beta = 0.5$

To further validate the LMM results, the temperature distributions for nucleate and radiation heat transfer modes are displayed specifically in Table 3 in comparisons with numerical results for $0.00 \le \xi \le 1.00$: Nucleate Boiling: (a) $m = 2, \beta = 0.1, \psi = 0.3$; (b) $m = 2, \beta = 0.5, \psi = 1.0$;Radiation: (c) $m = 3, \beta = 0.1, \psi = 0.3$; (d) $m = 3, \beta = 0.5, \psi = 1.0$. It is clearly seen that the LMM agrees exactly with the numerical experiments, thereby demonstrating the efficiency of the LMM.Of course, there is no gaining that the respective roles of the thermo-geometric ψ and thermal conductivity β parameters are equally convincingly characterized by the results. It is worthy to note that the temperature distribution increases with increasing dimensionless distance ξ of the fin with the thermal radiation heat transfer playing a dominating role as compared to the nucleate boiling heat transfer. This demonstrates, once again, the fact that the engineering applications of fins depend on the specific environmental needin relation to the choice of the heat transfer mode, taking cognizance of the respective roles of the thermo-geometric and thermal conductivity parameters. A typical scenario is that the design of some electrical appliances used in temperate regions may not necessarily be the same with those used in cold regions. For example, some electronics and electrical appliances are specifically designed for Nigeria and other African Countries, which are different from those designed to be used in Europe due to the great temperature difference year in, year out.

ξ	(a)Nucleate Boiling, $m = 2, \beta = 0.1, \psi = 0.3$		(b)Nucleate Boiling, $m = 2, \beta = 0.5, \psi = 1.0$		(c)Radiation, $m = 3, \beta = 0.1, \psi = 0.3$		(d)Radiation, $m = 3, \beta = 0.5, \psi = 1.0$	
	LMM	**NUM**	**LMM**	**NUM**	**LMM**	**NUM**	**LMM**	**NUM**
0.00	0.962724	0.962724	0.800260	0.800260	0.963755	0.963755	0.820310	0.820310
0.05	0.962815	0.962815	0.800718	0.800718	0.963843	0.963843	0.820712	0.820712

0.10	0.963090	0.963090	0.802092	0.802092	0.964109	0.964109	0.821917	0.821917
0.15	0.963548	0.963548	0.804386	0.804386	0.964552	0.964552	0.823931	0.823931
0.20	0.964190	0.964190	0.807605	0.807605	0.965172	0.965172	0.826759	0.826759
0.25	0.965015	0.965015	0.811757	0.811757	0.965971	0.965971	0.830409	0.830409
0.30	0.966025	0.966025	0.816854	0.816854	0.966948	0.966948	0.834894	0.834894
0.35	0.967220	0.967220	0.822907	0.822907	0.968105	0.968105	0.840228	0.840228
0.40	0.968600	0.968600	0.829932	0.829932	0.969441	0.969441	0.846428	0.846428
0.45	0.970167	0.970167	0.837948	0.837948	0.970958	0.970958	0.853514	0.853514
0.50	0.971920	0.971920	0.846975	0.846975	0.972658	0.972658	0.861510	0.861510
0.55	0.973862	0.973862	0.857037	0.857037	0.974541	0.974541	0.870445	0.870445
0.60	0.975992	0.975992	0.868162	0.868162	0.976608	0.976608	0.880349	0.880349
0.65	0.978313	0.978313	0.880380	0.880380	0.978862	0.978862	0.891258	0.891258
0.70	0.980825	0.980825	0.893727	0.893727	0.981303	0.981303	0.903214	0.903214
0.75	0.983530	0.983530	0.908241	0.908241	0.983933	0.983933	0.916262	0.916262
0.80	0.986429	0.986429	0.923964	0.923964	0.986755	0.986755	0.930457	0.930457
0.85	0.989523	0.989523	0.940945	0.940945	0.989770	0.989770	0.945856	0.945856
0.90	0.992815	0.992815	0.959238	0.959238	0.992981	0.992981	0.962527	0.962527
0.95	0.996307	0.996307	0.978901	0.978901	0.996390	0.996390	0.980546	0.980546
1.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table 3: Comparisons of LMM and Numerical solution for (a) Nucleate Boiling, $m = 2, \beta = 0.1,\ \psi = 0.3$; (b) Nucleate Boiling, $m = 2, \beta = 0.5,\ \psi = 1.0$; (c) Radiation, $m = 3, \beta = 0.1,\ \psi = 0.3$; (d) Radiation, $m = 3, \beta = 0.5,\ \psi = 1.0$.

6. CONCLUDING REMARKS

The nonlinear problem of convective straight fins with temperature-dependent thermal conductivity and heat transfer coefficient has been tackled by the use of Leibnitz-Maclaurin Method (LMM) via

Successive Differential Coefficients (SDC), which is a power series solution technique. Also, an exact implicit integral solutionwas constructed for the problem. The obtained solutions satisfy the physical boundary conditions. Benchmarks or validation tests of the LMM and the exact implicit integral solution together with numerical experiments demonstrated excellent agreements. The main conclusions are as follows:

1) The temperature θ_w at the tip of the fin increases with increase in the heat transfer mode m.
2) The thermal conductivity β and the thermo-geometric ψ parameters enhances the heat transfer of the fin, with β playing a dominant role.
3) The thermo-geometric parameter physically accounts for convective-conductive effects in the fin system in the absence of thermal conductivity.
4) The fin tip temperature decreases to zero when the magnitude of the fin base gradient temperature $\theta'(1)$ increases, and the rate of decay to zero decreases with increasing fin base gradient when the value of m decreases.
5) The thermo-geometric parameter enhances the fin base gradient temperature at the absence of the thermal conductivity parameter
6) The presence of the thermal conductivity parameter decreases the fin base gradient temperature as the heat transfer mode increases.
7) . The fin efficiency η decreases with increasing ψ or m for $m > -1$, while the value of β increases the layer of ψ.
8) The variability of the thermal conductivity and heat transfer coefficientdemonstrated the physical usefulness and serves as a practical departure from the previously investigated constant scenarios of thermal conductivity and heat transfer coefficient.
9) The LMM demonstrated high accuracy and excellent performance, characterizing its convergence to the exact implicit integral solution, simplicity, algorithmic nature, and not requiring any linearization, discretization, or perturbation.

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8. REFERENCES

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